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MILLOUX INEQUALITY OF NONLINEAR DIFFERENCE MONOMIALS AND ITS APPLICATION

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Abstract. Let $f(z)$ be a transcendental meromorphic function of finite order and $c_1, c_2, \cdots, c_m$ be complex constants satisfying that at least one of them is non-zero. The authors establish an inequality (Milloux inequality) about the nonlinear difference monomials $f^{d_1}(z+c_1)f^{d_2}(z+c_2)\cdots f^{d_m}(z+c_m)$, where $d_1, d_2, \cdots, d_m \in \mathbb{N}$. As an application of the inequality, the authors investigate the value distribution of $f^{d_1}(z+c_1)f^{d_2}(z+c_2)\cdots f^{d_m}(z+c_m)$. Results obtained partially promote and improve relevant results of Laine, Yang and Chen et al..

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