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COORDINATE STRONGLY s–CONVEX FUNCTIONS AND RELATED RESULTS

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Abstract. In this article, we give non-trivial examples of coordinate s-convex functions which are not s-convex functions. Also, we present a new class of coordinate strongly s-convex functions. We prove that every strongly s-convex function is coordinate strongly s-convex function but the converse is not generally true. Furthermore, we establish Jensen type inequality for strongly s-convex functions. We present Jensen and Hermite-Hadamard type inequalities for coordinate strongly s-convex functions.

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