ON THE REFINEMENT OF QUANTUM HERMITE–HADAMARD INEQUALITIES FOR CONTINUOUS CONVEX FUNCTIONS

JULALAK PRABSEANG, KAMSING NONLAOPON AND SOTIRIS K. NTOUYAS

Abstract. The purpose of this paper is to establish some new refinement of quantum Hermite-Hadamard inequalities for continuous convex functions. Several known results are reduced as special cases.

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