# BOUNDS FOR THE PERIMETER OF AN
# ELLIPSE IN TERMS OF POWER MEANS

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*Abstract.* In the article, we provide several precise bounds for the perimeter of an ellipse in terms of the power means, and present new bounds for the complete elliptic integral of the second kind. The given results are the improvements of some earlier results.

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