REAL INTERPOLATION WITH A FUNCTION PARAMETER FOR MARTINGALE HARDY–LORENTZ AND BMO SPACES

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Abstract. This paper is devoted to the study of real interpolation between martingale Hardy-Lorentz and BMO spaces in the framework of interpolation with a function parameter. We first establish some inequalities for the sharp functions of martingales. With the aid of these inequalities, some new interpolation theorems which generalize some fundamental interpolation theorems in classical martingale H_p theory are proved. In particular, we show that

$$(H^s_{p_0,q_0}, BMO_2)_{\rho,q} = \Lambda^s_q(t^{\frac{1}{p_0}}/\rho(t^{\frac{1}{p_0}})),$$

where $0 < p_0 < \infty$, $0 < q_0, q \leqslant \infty$ and $\rho \in Q(0,1)$.

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