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ASYMPTOTIC BOUNDS FOR PRECISE LARGE DEVIATIONS IN A COMPOUND RISK MODEL UNDER DEPENDENCE STRUCTURES

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Abstract. In the paper, we consider a compound risk model, where all the claim sizes satisfy a dependence structure, and the accident inter-arrival time and the claim-number of the subsequent accident satisfy another dependence structure described by a conditional tail probability of the inter-arrival time given the subsequent claim-number. We obtain the asymptotic lower and upper bounds for the precise large deviations of the aggregate claims, with a feature that the asymptotic bounds hold uniformly for all x in an infinite t-interval.

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