Journal of
Mathematical
Inequalities
Volume 14, Number 4 (2020), 1113–1122 doi:10.7153/jmi-2020-14-72

HARNACK INEQUALITY FOR STOCHASTIC HEAT EQUATION DRIVEN BY FRACTIONAL NOISE WITH HURST INDEX $H > \frac{1}{2}$

XIUWEI YIN, GUANGJUN SHEN* AND ZHENLONG GAO

Abstract. In this short note, we establish the dimensional-free Harnack inequality for stochastic heat equation with Dirichlet boundary condition:

$$\begin{cases} \frac{\partial}{\partial t}u(t,x) = \frac{\partial^2}{\partial x^2}u(t,x) + b(u(t,x)) + \dot{W}^H(t,x),\ 0 < t \leqslant T,\ 0 < x < 1, \\ u(t,0) = u(t,1) = 0,\ 0 < t \leqslant T, \\ u(0,x) = f(x),\ 0 \leqslant x \leqslant 1, \end{cases}$$

where $T > 0$, $f(x) \in L^2([0,1])$ and $W^H(t,x)$ is the fractional noise with Hurst index $H \in (\frac{1}{2}, 1)$. The strong Feller property is also obtained.

Mathematics subject classification (2010): 60H15, 60H35, 65C30.
Keywords and phrases: Stochastic heat equation, fractional noise, Harnack inequality.

REFERENCES

[1] S. AIDA, H. KAWABI, *Short time asymptotics of a certain infinite dimensional diffusion process*, in: Stochastic Analysis and Related Topics, VII, Kusadasi, 1998, in: Progr. Probab., vol. 48, Boston, MA, 2001, 77–124.

[2] S. AIDA, T. ZHANG, *On the small time asymptotics of diffusion processes on path groups*, Potential Anal. **16** (2002), 67–78.

[3] S. BOBKOV, I. GENTIL, M. LEDOUX, *Hypercontractivity of Hamilton-Jacobi equations*, J. Math. Pures Appl. **80** (2001), 669–696.

[4] L. DECREUSEFOND AND A. S. ÜSTÜNEL, *Stochastic analysis of the fractional Brownian motion*, Potential Anal. **10** (1998), 177–214.

[5] O. EL BARRIMI, Y. OUKNINE, *Stochastic differential equations driven by an additive fractional Brownian sheet*, Bull. Korean Math. Soc. **56 (2)** (2019), 479–489.

[6] X. L. FAN, *Harnack inequality and derivative formula for SDE driven by fractional Brownian motion*, Sci. China Ser. A. **561** (2013), 515–524.

[7] X. L. FAN, *Harnack-type inequalities and applications for SDE driven by fractional Brownian motion*, Stoch. Anal. Appl. **32** (2014), 602–618.

[8] F. GONG, F. Y. WANG, *Heat kernel estimates with application to compactness of manifolds*, Q. J. Math. **52** (2001), 171–180.

[9] A. HARNACK, *Die Grundlagen der Theorie des logarithmischen Potentiales und der eindeutigen Potentialfunktion in der Ebene*, V. G. Teubner, Leipzig, 1887.

[10] X. HUANG, *Harnack and shift Harnack inequalities for SDEs with integrable drifts*, Stoch. Dyn. **19 (5)** (2019), 1950034.

[11] D. NUALART, Y. OUKNINE, *Regularization of quasilinear heat equations by a fractional noise*, Stoch. Dyn. **4** (2004), 201–221.

[12] M. RÖCKNER, F. Y. WANG, *Harnack and functional inequalities for generalized Mehler semigroups*, J. Funct. Anal. **203** (2003), 237–261.

[13] M. RÖCKNER, F. Y. WANG, *Supercontractivity and ultracontractivity for (non-symmetric) diffusion semigroups on manifolds*, Forum Math. **15** (2003), 893–921.

Paper JMI-14-72

[14] F. Y. WANG, C. YUAN, *Harnack inequalities for functional SDEs with multiplicative noise and applications*, Stochastic Process. Appl. **121** (2011), 2692–2710.

[15] F. Y. WANG, J. WANG, *Harnack inequalities for stochastic equations driven by Lévy noise*, J. Math. Anal. Appl. **410** (2014) 513–523.

[16] F. Y. WANG, *Harnack Inequality and Applications for Stochastic Partial Differential Equations*, Springer-Verlag, 2013.

[17] F. Y. WANG, *Logarithmic Sobolev inequalities on noncompact Riemannian manifolds*, Probab. Theory Related Fields, **109** (1997), 417–424.

[18] L. T. YAN, X. W. YIN, *Harnack inequality and derivative formula for stochastic heat equation with fractional noise*, Electron. Comm. Probab. **23 (35)** (2018), 1–11.

[19] L. T. YAN, X. W. YIN, *Bismut formula for a stochastic heat equation with fractional noise*, Statist. Probab. Lett. **137** (2018), 165–172.

[20] S. Q. ZHANG, *Harnack inequality for semilinear SPDEs with multiplicative noise*, Statist. Probab. Lett. **83** (2013), 1184–1192.

[21] T. S. ZHANG, *White noise driven SPDEs with reflection: strong Feller properties and Harnack inequalities*, Potential Anal. **33** (2010), 137–151.

[22] F. Y. WANG, *Harnack inequalities for log-Sobolev functions and estimates of log-Sobolev constants*, Ann. Probab. **27** (1999), 653–663.

Journal of Mathematical Inequalities
www.ele-math.com
jmi@ele-math.com