

THE PARTICULAR SOLUTION AND ULAM STABILITY OF LINEAR RIEMANN–LIOUVILLE FRACTIONAL DYNAMIC EQUATIONS ON ISOLATED TIME SCALES

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Abstract. By using the Laplace transform method on isolated time scales, this paper deals with the particular solution and the Ulam stability of linear Riemann-Liouville fractional dynamic equations with constant coefficients.

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