

INEQUALITIES FOR GAUSSIAN HYPERGEOMETRIC FUNCTIONS

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Abstract. In this paper, the authors present several hypergeometric transformation inequalities for the Gaussian hypergeometric function $F(a, b; c; x)$, which are the extensions of the known hypergeometric transformation identities such as Ramanujan's cubic transformation identities, by showing the monotonicity properties of certain quotients of $F(a, b; c; x)$ and its special cases. By these results, some related known results are considerably improved.

Mathematics subject classification (2010): 33C05, 39B62.

Keywords and phrases: Gaussian hypergeometric function, Ramanujan's transformation, infinite product.

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