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WEIGHTED INTEGRAL INEQUALITY AND APPLICATION IN UNIFORM STABILITY FOR A NONLINEAR SYSTEM WITH MEMORY

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Abstract. In this paper, we consider the viscoelastic system

$$\boldsymbol{u}_{tt} - \mu\Delta\boldsymbol{u} - (\lambda+\mu)\nabla(\mathrm{div}\boldsymbol{u}) + \int_0^t g(t-s)\mathrm{div}[a(x)\nabla\boldsymbol{u}(s)]ds + b(x)\boldsymbol{h}(\boldsymbol{u}_t) = \boldsymbol{f}(\boldsymbol{u})$$

with initial conditions and boundary conditions. Under some assumptions on the relaxation function g, and other functions $\boldsymbol{h}$ and $\boldsymbol{f}$, without constructing any auxiliary functional, by establishing weighted integral inequality on the energy functional, we obtain a general energy decay formula for the solution, such that the usual exponential decay results and the polynomial decay results are only special cases, respectively.

Mathematics subject classification (2010): 35L05, 35L15, 35L70.
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