

A NOTE ON VARIANCE BOUNDS AND LOCATION OF EIGENVALUES

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Abstract. We discuss some extensions and refinements of the variance bounds for both real and complex numbers. The related bounds for the eigenvalues and spread of a matrix are also derived here.

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