A NEW FRACTIONAL BOUNDARY VALUE
PROBLEM AND LYAPUNOV–TYPE INEQUALITY

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Abstract. Throughout this paper, we study a new modified version of fractional boundary value problem (BVP) of the form

$$(_a^C D^\alpha y)(t) + p(t)y'(t) + q(t)y(t) = 0, \qquad a < t < b, \qquad 2 < \alpha \leqslant 3,$$

with $y(a) = y'(a) = y(b) = 0$, where $p \in C^1([a,b])$ and $q \in C([a,b])$. Using the vector Green function we obtain a Lyapunov-type inequality for the BVP subject to Dirichlet-type boundary conditions. Moreover, we utilize the new inequality to infer a criteria for the nonexistence of real zeros of some certain Mittag-Leffler functions using the generalized Wright functions.

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