DERIVATION–HOMOMORPHISM FUNCTIONAL INEQUALITIES

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Abstract. In this paper, we introduce and solve the following additive-additive \((s,t)\)-functional inequality

\[
\|g(x + y) - g(x) - g(y)\| + \|h(x + y) + h(x) - 2h(x)\| \\ \leq \left| s \left( 2g \left( \frac{x + y}{2} \right) - g(x) - g(y) \right) \right| + \left| t \left( 2h \left( \frac{x + y}{2} \right) + 2h \left( \frac{x - y}{2} \right) - 2h(x) \right) \right|,
\]

where \(s\) and \(t\) are fixed nonzero complex numbers with \(|s| < 1\) and \(|t| < 1\). Using the direct method and the fixed point method, we prove the Hyers-Ulam stability of derivation-homomorphisms in complex Banach algebras, associated to the additive-additive \((s,t)\)-functional inequality (1) and the following functional inequality

\[
\|g(xy) - g(x)y - xg(y)\| + \|h(xy) - h(x)h(y)\| \leq \varphi(x,y).
\]


Keywords and phrases: Hyers-Ulam stability, direct method, fixed point method, additive-additive \((s,t)\)-functional inequality, derivation in Banach algebra, homomorphism in Banach algebra.

REFERENCES