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DERIVATION–HOMOMORPHISM FUNCTIONAL INEQUALITIES

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Abstract. In this paper, we introduce and solve the following additive-additive (s,t)-functional inequality

$$\begin{aligned} &\|g(x+y)-g(x)-g(y)\|+\|h(x+y)+h(x-y)-2h(x)\| \\ &\leqslant \left\|s\left(2g\left(\frac{x+y}{2}\right)-g(x)-g(y)\right)\right\|+\left\|t\left(2h\left(\frac{x+y}{2}\right)+2h\left(\frac{x-y}{2}\right)-2h(x)\right)\right\|, \end{aligned} \tag{1}$$

where s and t are fixed nonzero complex numbers with $|s|<1$ and $|t|<1$. Using the direct method and the fixed point method, we prove the Hyers-Ulam stability of derivation-homomorphisms in complex Banach algebras, associated to the additive-additive (s,t)-functional inequality (1) and the following functional inequality

$$\|g(xy)-g(x)y-xg(y)\|+\|h(xy)-h(x)h(y)\| \leqslant \varphi(x,y). \tag{2}$$

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