SELF–ADAPTIVE ALGORITHMS FOR AN
EQUILIBRIUM SPLIT PROBLEM IN HILBERT SPACES

WENLONG SUN*, GANG LU, YUANFENG JIN* AND CHOONKIL PARK

Abstract. In this paper, we propose and study iterative algorithms for solving the split problem: find a common element $x^\dagger \in C$ satisfying

$$\Theta(x^\dagger, y) + \langle Fx^\dagger, y - x^\dagger \rangle + \psi(x^\dagger, y) - \psi(x^\dagger, x^\dagger) \geqslant 0, \quad \forall y \in C$$

and

$$Au \in Fix(S),$$

where S be an L-Lipschitzian quasi-pseudo-contractive operator. Weak and strong convergence theorems are given under some mild assumptions.

Mathematics subject classification (2020): Primary 47H06, 47H09, 49J05, 47J25.

Keywords and phrases: Equilibrium split problem, quasi-pseudo-contractive operator, self-adaptive algorithm.

REFERENCES

[1] S. H. WANG, X. M. LIU AND Y. S. AN, *A new iterative algorithm for generalized split equilibrium problem in Hilbert spaces*, Nonlinear Func. Anal. Appl., 22 (4) (2017), 911–924.

[2] H. H. BAUSCHKE, *Combettes, P. L. Convex Analysis and Monotone Operator Theory in Hilbert Spaces*, Springer, New York, NY, USA, 2011.

[3] X. P. DING, *Auxiliary principle and algorithm for mixed equilibrium problems and bilevel mixed equilibrium problems in Banach Spaces*, J. Optim. Theory Appl. 146, 347–357 (2010).

[4] K. R. KAZMI, A. KHALIQ, A. RAOUF, *Iterative approximation of solution of generalized mixed set-valued variational inequality problem*, Math. Inequal. Appl. 10 (3), 677–691 (2007).

[5] K. R. KAZMI, S. YOUSUF, *Common solution to generalized mixed equilibrium problem and fixed point problems in Hilbert space*, RACSAM 113, 3699–3715 (2019),
https://doi.org/10.1007/s13398-019-00725-1.

[6] N. ONJAI-UEA, W. PHUENGRATTANA, *On solving split mixed equilibrium problems and fixed point problems of hybrid-type multivalued mappings in Hilbert spaces*, J. Inequal Appl. 2017; 2017 (1): 137, doi:10.1186/s13660-017-1416-x.

[7] E. BLUM, W. OETTLI, *From optimization and variational inequalities to equilibrium problems*, Math. Student 63 (1994) 123–145.

[8] K. GOEBEL, W. A. KIRK, *Topics in metric fixed point theory*, Cambridge: Cambridge Univ Press; 1990, (Cambridge studies in advanced mathematics).

[9] A. MOUDAFI, *Second order differential proximal methods for equilibrium problems*, J. Inequal. Pure Appl. Math. 4 (1) (2003), Art. 18.

[10] Y. YAO, Y. J. CHO, Y. C. LIOU, *Algorithms of common solutions for variational inclusions, mixed equilibrium problems and fixed point problems*, European Journal of Operational Research, 2011, 212 (2): 242–250.

[11] M. A. NOOR, *Auxiliary principle technique for equilibrium problems*, J. Optim. Theory Appl. 122, 371–386 (2004).

[12] A. MOUDAFI, M. THRA, *Proximal and dynamical approaches to equilibrium problems*, Lecture Notes in Economics and Mathematical Systems, vol. 477, pp. 187–201, Springer, New York (1999).

[13] A. Moudafi, *Mixed equilibrium problems: Sensitivity analysis and algorithmic aspect*, Comput. Math. Appl. 44, 1099–1108 (2002).

[14] D. V. Hieu, *New extragradient method for a class of equilibrium problems in Hilbert spaces*, Appl. Anal. 97, 811–824 (2018).

[15] D. V. Hieu, *New subgradient extragradient methods for common solutions to equilibrium problems*, Comput. Optim. Appl. 67, 571–594 (2017).

[16] D. V. Hieu, *An extension of hybrid method without extrapolation step to equilibrium problems*, J. Ind. Manag. Optim. 13, 1723–1741 (2017).

[17] D. V. Hieu, *Halpern subgradient extragradient method extended to equilibrium problems*, Rev. R. Acad.

[18] K. R. Kazmi, S. H. Rizvi, *An iterative algorithm for generalized mixed equilibrium problem*, Afr. Mat. 25, 857–867 (2014).

[19] A. S. Antipin, *Iterative gradient predictor type methods for computing fixed point of external mappings*, in: Guddat, J., Jonden, H.Th., Nizicka, F., Still, G., Twitt F. (eds.) Parametric optimization and related topics IV[C], Peter Lang, Frankfurt, Main, pp. 11–24 (1997).

[20] Q. H. Ansari, A. Rehan, and C. F. Wen, *Implicit and explicit algorithms for split common fixed point problems*, J. Nonlinear Convex Anal., 17 (2016), 1381–1397.

[21] A. E. Al-Mazrooei, A. Latif, X. Qin, et al. *Fixed point algorithms for split feasibility problems*, Fixed Point Theory, 2019; 20 (1): 245–254.

[22] O. A. Boikanyo, *A strongly convergent algorithm for the split common fixed point problem*, Appl. Math. Comput., 265 (2015), 844–853.

[23] N. Buong, *Iterative algorithms for the multiple-sets split feasibility problem in Hilbert spaces*, Numer. Algorithms 2017; 76: 783–798.

[24] Y. Censor, T. Elfving, *A multiprojection algorithm using Bregman projections in a product space*, Numer. Algorithms 8, 221–239 (1994).

[25] Y. Censor, T. Elfving, N. Kopf, et al., *The multiple-sets split feasibility problem and its applications for inverse problem*, Inverse Probl. 2005; 21: 2071–2084.

[26] A. Cegielski, *General method for solving the split common fixed point problem*, J. Optim. Theory Appl. 2015; 165: 385–404.

[27] Y. Censor, A. Segal, *The split common fixed point problem for directed operators*, J. Convex Anal. 2009; 16: 587–600.

[28] S. He, Z. Zhao, B. Luo, *A relaxed self-adaptive CQ algorithm for the multiple-sets split feasibility problem*, Optimization 2015; 64: 1907–1918.

[29] P. Kraikaew, S. Saejung, *On split common fixed point problems*, J. Math. Anal. Appl. 2014; 415: 513–524.

[30] P. E. Maing, *Strong convergence of projected reflected gradient methods for variational inequalities*, Fixed Point Theory 2018; 19: 659–680.

[31] A. Moudafi, *The split common fixed-point problem for demicontractive mappings*, Inverse Probl. 2010; 26: 055007.

[32] W. Takahashi, *The split common fixed point problem and strong convergence theorems by hybrid methods in two Banach spaces*, J. Nonlinear Convex Anal. 2016; 17: 1051–1067.

[33] W. Takahashi, C. F. Wen, J. C. Yao, *An implicit algorithm for the split common fixed point problem in Hilbert spaces and applications*, Appl. Anal. Optim. 2017; 1: 423–439.

[34] F. Wang, *A new method for split common fixed-point problem without priori knowledge of operator norms*, J. Fixed Point Theory Appl., doi:10.1007/s11784-017-0434-0.

[35] H. K. Xu, *Iterative algorithms for nonlinear operators*, J. Lond. Math. Soc. 66, 240–256 (2002).

[36] Y. Yao, Y.-C. Liou, J.-C. Yao, *Split common fixed point problem for two quasi-pseudocontractive operators and its algorithm construction*, Fixed Point Theory Appl. 2015, 2015, 127.

[37] Y. H. Yao, Y. C. Liou, M. Postolache, *Self-adaptive algorithms for the split problem of the demicontractive operators*, Optimization 2018; 67: 1309–1319.

[38] Y. H. Yao, Y. C. Yao, Y. C. Liou, et al., *Iterative algorithms for split common fixed points of demicontractive operators without priori knowledge of operator norms*, Carpathian J. Math. 2018; 34: 459–466.

Journal of Mathematical Inequalities
www.ele-math.com
jmi@ele-math.com