A NEW ALGORITHM FOR FINDING A COMMON SOLUTION OF A SPLIT VARIATIONAL INEQUALITY PROBLEM, THE FIXED POINT PROBLEMS AND THE VARIATIONAL INCLUSION PROBLEMS

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Abstract. In this paper, we present a new iterative algorithm for finding a common element of the set of solutions of a split variational inequality problem, the set of fixed points of an infinite family of nonexpansive mappings and the set of solutions of a variational inclusion problem in Hilbert spaces. Under some mild conditions imposed on algorithm parameters, we prove that the proposed iterative algorithm have strong convergence.

REFERENCES

[1] H. H. BAUSCHKE, P. L. COMBETTES, *Convex Analysis and Monotone Operator Theory in Hilbert Spaces*, Springer, New York (2011).

[2] W. TAKAHASHI, *Introduction to Nonlinear and Convex Analysis*, Yokohama Publishers, Yokohama (2009).

[3] S. M. ROBINSON, *Generalized equation and their solutions*, part I: Basic theory, Mathematics Programs of Study 10 (1979) 128–141.

[4] R. T. ROCKAFELLAR, *Monotone operators and the proximal point algorithm*, SIAM Journal on Control and Optimization 14 (1976) 877–898.

[5] K. SHIMOJI, W. TAKAHASHI, *Strong convergence to common fixed points of infinite nonexpasnsive mappings and applications*, Taiwanese Journal of Mathematics 5 (2001) 387–404.

[6] R. U. VERMA, *General system of (A,g)-monotone variational inclusion problems based on generalized hybrid iterative algorithm*, Nonlinear Analysis: Hybrid Systems 1 (2007) 326–335.

[7] R. AHMAD, J. IQBAL, S. AHMED AND S. HUSAIN, *Solving a variational inclusion problem with its corresponding resolvent equation problem involving XOR-Operation*, Nonlinear Funct. Anal. and Appl., 24 (3) (2019), 691–713, doi.org/10.22771/nfaa.2019.24.03.10.

[8] K. AFASSINOU, O. K. NARAIN AND O. E. OTUNUGA, *Iterative algorithm for approximating solutions of split monotone variational inclusion, variational inequality and fixed point problems in real Hilbert spaces*, Nonlinear Func. Anal. Appl., 25 (3) (2020), 491–510, doi.org/10.22771/nfaa.2020.25.03.06.

[9] K. GOEBEL, W. A. KIRK, *Topics in metric fixed point theory*, Cambridge: Cambridge Univ Press, 1990. (Cambridge studies in advanced mathematics).

[10] S. S. ZHANG, H. W. LEE JOSEPH, C. K. CHAN, *Algorithms of common solutions for quasi variational inclusion and fixed point problems*, Applied Mathematics and Mechanics (English Edition) 29 (2008) 571–581.

[11] J. W. PENG, Y. WANG, D. S. SHYU, J. C. YAO, *Common solutions of an iterative scheme for variational inclusions, equilibrium problems and fixed point problems*, Journal of Iequalities and Applications 2008 (2008) 15, Article ID 720371.

[12] H. ZHOU, Y. ZHOU, G. FENG, *Iterative methods for solving a class of monotone variational inequality problems with applications*, J. Inequal. Appl. 2015, 68 (2015).

[13] G. M. Korpelevich, *An extragradient method for finding saddle points and for other problems*, Ékon. Mat. Metody 12, 747–756 (1976).

[14] G. Stampacchia, *Formes bilineaires coercivites surles ensembles convexes*, C. R. Acad. Sci. 1964, 258, 4413–4416.

[15] B.-F. Svaiter, *A class of Fejér convergent algorithms, approximate resolvents and the hybrid proximal extragradient method*, J. Optim. Theory Appl. 2014, 162, 133–153.

[16] Y. Yao, Y.-C. Liou, S.-M. Kang, *Approach to common elements of variational inequality problems and fixed point problems via a relaxed extragradient method*, Comput. Math. Appl. 2010, 59, 3472–3480.

[17] J. S. Jung, *Strong convergence of general iterative algorithms for pseudocontractive mappings in Hilbert spaces*, Nonlinear Func. Anal. Appl., 24 (2) (2019), 389–406, doi.org/10.22771/nfaa.2019.24.02.10.

[18] J. K. Kim, P. N. Anh and T. T. H. Anh and N. D. Hien, *Projection methods for solving the variational inequalities involving unrelated nonexpansive mappings*, J. Nonlinear and Convex Analysis, 21 (11) (2020), 2517–2537.

[19] K. Muangchoo, *A viscosity type projection method for Solving pseudomonotone variational inequalities*, Nonlinear Funct. Anal. and Appl., 26 (2) (2021), 347–371, doi.org/10.22771/nfaa.2021.26.02.08.

[20] F. Qi, D. Lim, B.-N. Guo, *Explicit formulas and identities for the Bell polynomials and a sequence of polynomials applied to differential equations*, Rev. R. Acad. Cienc. Exactas Fís. Nat. Ser. A Mat. 2018, in press.

[21] F. Qi, D.-W. Niu, B.-N. Guo, *Some identities for a sequence of unnamed polynomials connected with the Bell polynomials*, Rev. R. Acad. Cienc. Exactas Fís. Nat. Ser. A Math. 2018, in press.

[22] M.-L. Ye, Y.-R. He, *A double projection method for solving variational inequalities without monotonicity*, Comput. Optim. Appl. 2015, 60, 141–150.

[23] Q. H. Ansari, A. Rehan and C. F. Wen, *Implicit and explicit algorithms for split common fixed point problems*, J. Nonlinear Convex Anal., 17 (2016), 1381–1397.

[24] Y. Yao, Y.-C. Liou, J.-C. Yao, *Split common fixed point problem for two quasi-pseudocontractive operators and its algorithm construction*, Fixed Point Theory Appl. 2015, 2015, 127.

[25] O. A. Boikanyo, *A strongly convergent algorithm for the split common fixed point problem*, Appl. Math. Comput., 265 (2015), 844–853.

[26] Y. Censor, A. Segal, *The split common fixed point problem for directed operators*, J. Convex Anal. 2009; 16: 587–600.

[27] S. Y. Cho, X. Qin, J.-C. Yao, et al., *Viscosity approximation splitting methods for monotone and nonexpansive operators in Hilbert spaces*, J Nonlinear Convex Anal. 2018; 19: 251–264.

[28] H. K. Xu, *Iterative algorithms for nonlinear operators*, J. Lond. Math. Soc. 66, 240–256 (2002).

[29] P. Kraikaew, S. Saejung, *On split common fixed point problems*, J Math Anal Appl. 2014; 415: 513–524.

[30] A. Moudafi, *The split common fixed-point problem for demicontractive mappings*, Inverse Probl. 2010; 26: 055007.

[31] W. Takahashi, *The split common fixed point problem and strong convergence theorems by hybrid methods in two Banach spaces*, J. Nonlinear Convex Anal. 2016; 17: 1051–1067.

[32] Y. H. Yao, Y. C. Liou, M. Postolache, *Self-adaptive algorithms for the split problem of the demicontractive operators*, Optimization. 2018; 67: 1309–1319.

[33] Yonghong Yao, Mihai Postolache, Zhichuan Zhu, *Gradient methods with selection technique for the multiplesets split feasibility problem*, Optimization, 2020; 69:2, 269–288.

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