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CERTAIN QUANTUM ESTIMATES ON THE PARAMETERIZED INTEGRAL INEQUALITIES AND THEIR APPLICATIONS

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Abstract. The present paper aims to study the parameterized inequalities of Hadamard–Simpson type for quantum integrals. By employing a quantum integral identity of multi-parameter, we establish novel inequalities for a class of q-differentiable mappings, which are related to s-(α,m)-convex mappings. Moreover, we acquire estimation-type results by considering the boundedness and the Lipschitz condition. As applications, we present two illustrative examples and several quantum integral inequalities for the special means.

Mathematics subject classification (2010): 05A30, 34A08, 26A33, 26D15.
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