

COMPLETE MONOTONICITY OF SOME FUNCTIONS INVOLVING k -DIGAMMA FUNCTION WITH APPLICATION

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Abstract. We present several complete monotonicity properties involving k -digamma function with single parameter. These established results provide a k -generalization for the known results obtained by Burić and Elezović in [5]. Finally, we give an application to the generalized Nielsen's β -function and pose two open problems.

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