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A GENERAL ADDITIVE FUNCTIONAL INEQUALITY AND DERIVATION IN BANACH ALGEBRAS

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Abstract. Using the fixed point method, we prove the Hyers-Ulam stability of homomorphisms in complex Banach algebras and complex Banach Lie algebras and also of derivations on complex Banach algebras and complex Banach Lie algebras for the general additive functional inequality $\|f(\alpha x-\beta y)-\alpha f(x)-\beta f(-y)\| \leqslant \|r(f(\alpha x+\beta y)-\alpha f(x)-\beta f(y)\|$, where r is a fixed nonzero complex number with $|r|<1$ and $\alpha,\beta \neq 0$.

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