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SHARP INEQUALITIES FOR HERMITIAN TOEPLITZ DETERMINANTS FOR STRONGLY STARLIKE AND STRONGLY CONVEX FUNCTIONS

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Abstract. Sharp upper and lower bounds are found of the second and third order Hermitian Toeplitz determinants for the classes of strongly starlike and strongly convex functions of order α ($\alpha \in [0,1)$).

Mathematics subject classification (2010): 30C45, 30C50.
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REFERENCES

[1] J. W. ALEXANDER, *Functions which map the interior of the unit circle upon simple regions*, Ann. of Math., **17**, (1915), 12–22.
[2] MD FIROZ ALI, D. K. THOMAS, A. VASUDEVARAO, *Toeplitz determinants whose elements are the coefficients of analytic and univalent functions*, Bull. Austr. Math. Soc., **97**, 2 (2018), 253–264.
[3] R. M. ALI, V. SINGH, *On the fourth and fifth coefficients of strongly starlike functions*, Results Math., **29**, (1996), 197–202.
[4] D. A. BRANNAN, W. E. KIRWAN, *On some classes of bounded univalent functions*, J. London Math. Soc., **2 (1)**, (1969), 431–443.
[5] C. CARATHÉODORY, *Über den Variabilitatsbereich der Koeffizienten von Potenzreihen, die gegebene werte nicht annehmen*, Math. Ann., **64**, (1907), 95–115.
[6] N. E. CHO, B. KOWALCZYK, O. S. KWON, A. LECKO, Y. J. SIM, *Some coefficient inequalities related to the Hankel determinant for strongly starlike functions of order alpha*, J. Math. Ineq., **11**, 2 (2017), 429–439.
[7] N. E. CHO, B. KOWALCZYK, A. LECKO, *Sharp bounds of some coefficient functionals over the class of functions convex in the direction of the imaginary axis*, Bull. Aust. Math. Soc., **100**, (2019), 86–96.
[8] K. CUDNA, O. S. KWON, A. LECKO, Y. J. SIM, B. ŚMIAROWSKA, *The second and third-order Hermitian Toeplitz determinants for starlike and convex functions of order* α, Boletin Soc. Mat. Mex., **26**, (2020), 361–375.
[9] A. W. GOODMAN, *Univalent Functions*, Mariner, Tampa, Florida, 1983.
[10] B. KOWALCZYK, O. S. KWON, A. LECKO, Y. J. SIM, B. ŚMIAROWSKA, *The third-order Hermitian Toeplitz determinant for classes of functions convex in one direction*, Bull. Malays. Math. Sci. Soc., **43**, (2020), 3143–3158.
[11] B. KOWALCZYK, A. LECKO, Y. J. SIM, *The sharp bound of the Hankel determinant of the third kind for convex functions*, Bull. Aust. Math. Soc., **97**, (2018), 435–445.
[12] O. S. KWON, A. LECKO, Y. J. SIM, B. ŚMIAROWSKA, *The sharp bound of the fifth coefficient of strongly starlike functions with real coefficients*, Bull. Malays. Math. Sci. Soc., **42**, 4 (2019), 1719–1735.
[13] A. LECKO, *Some Methods in the Theory of Univalent Functions*, Oficyna Wydawnicza Politechniki Rzeszowskiej, Rzeszów, 2005.
[14] A. LECKO, *Strongly starlike and spirallike functions*, Ann. Polon. Math., **85**, 2 (2005), 165–192.
[15] W. MA, D. MINDA, *An internal geometric characterization of strongly starlike functions*, Ann. Univ. Mariae Curie Skłodowska Sect. A, **20**, (1991), 89–97.
[16] W. MA, S. OWA, *Strongly starlike functions*, Panam. Math. J., **3**, 2 (1993), 49–60.

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[17] R. NEVANLINNA, *Über die konforme Abbildung von Sterngebieten*, Översikt av Finska Vetens.-Soc. Förh., Avd. A, **LXIII**, 6 (1920–1921), 1–21.
[18] C. POMMERENKE, *Univalent functions*, Vandenhoeck & Ruprecht, Göttingen, 1975.
[19] E. STUDY, *Vorlesungen über ausgewählte Gegenstände der Geometrie, Zweites Heft; Konforme Abbildung Einfach-Zusammenhängender Bereiche*, Druck und Verlag von B. G. Teubner, Leipzig und Berlin, 1913.
[20] J. STANKIEWICZ, *Quelques problèmes extrémaux dans les classes des fonctions α-angulairement étoilées*, Ann. Univ. Mariae Curie-Skłodowska Sect. A, **20**, (1966), 59–75.
[21] J. STANKIEWICZ, *On a family of starlike functions*, Ann. Univ. Mariae Curie-Skłodowska Sect. A, **22–24**, (1968–1970), 175–181.
[22] T. SUGAWA, *A self-duality of strong starlikeness*, Kodai Math. J., **28**, (2005), 382–389.