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ON AN EQUATION CHARACTERIZING MULTI–JENSEN–QUARTIC MAPPINGS AND ITS STABILITY

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Abstract. In this paper, we introduce a new form of the multi-quartic mappings and then unify the system of functional equations defining a multi-Jensen-quartic mapping to a single equation. Applying a fixed point theorem, we study the generalized Hyers-Ulam stability of multi-Jensen-quartic mappings. We present a few corollaries corresponding to some known stability outcomes on the multi-quartic and the multi-Jensen-quartic functional equations.

Mathematics subject classification (2010): 39B52, 39B72, 39B82.
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