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AN INEQUALITY ON THE DIFFERENCE POLYNOMIALS OF MEROMORPHIC FUNCTIONS AND ITS APPLICATION

ZHAOJUN WU

Abstract. Let $f(z)$ be a transcendental meromorphic function of finite order and $\Psi(z) = f(z+c_1)f(z+c_2)\cdots f(z+c_n) - a(f(z))^n$ be a difference polynomials of f, where $a \in \mathbb{C}\setminus\{0\}$, $c_1, c_2, \cdots, c_n (n \in \mathbb{N}^+)$ be complex constants satisfying that at least one of them is non-zero. If $\Psi(z)$ is transcendental, the author establishes the following inequality on $\Psi(z)$:

$$nT(r,f) \leqslant nN\left(r,\frac{1}{f}\right) + 4nN(r,f) + N\left(r,\frac{1}{\Psi(z)-b}\right) + S(r,f),$$

where $b \in \mathbb{C}\setminus\{0\}$. As an application of this inequality, the author investigates the value distribution of $\Psi(z)$. Results are obtained partially solve some open questions raised by Zheng and Chen in [X. M. Zheng, Z. X. Chen, On the value distribution of some difference polynomials, J. Math. Anal. Appl. 397(2013) 814–821].

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