Journal of Mathematical Inequalities
Volume 15, Number 1 (2021), 357–374
doi:10.7153/jmi-2021-15-27

APPROXIMATION PROPERTIES OF COMBINATION OF MULTIVARIATE AVERAGES ON TRIEBEL–LIZORKIN SPACES

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Abstract. The purpose of this paper is to establish the rate of approximation of the combination of some generalized multivariate average on Triebel-Lizorkin spaces and obtain its equivalent relation to the K-functionals. These results significantly generalize some known results in the literatures.

Mathematics subject classification (2010): 41A17, 42B35.
Keywords and phrases: K-functional, Triebel-Lizorkin space, multivariate average.

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