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MAPPING PROPERTIES OF MULTILINEAR FRACTIONAL MAXIMAL OPERATORS IN METRIC MEASURE SPACES

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Abstract. In this paper, we introduce two kinds of multilinear fractional maximal operators in metric measure spaces. We prove that these operators map product Morrey spaces to Morrey spaces, and map product Lebesgue spaces to the fractional Hajłasz spaces under certain restrictions on the underlying metric measure space. We also introduce a kind of discrete multilinear fractional maximal operator, which is constructed in terms of coverings and partitions of unities and has better regularity. With the aid of Poincaré inequality, we establish the Sobolev bounds for the above operators.

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