A REVERSE BLASCHKE–SANTALÓ INEQUALITY

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Abstract. Motivated by works of Lutwak, Yang and Zhang (in [17], [20] and [21]), a new reverse Blaschke-Santaló inequality that connects the volume of an origin-symmetric convex body K with the volume of the polar body $\Gamma_2^* K$ of the L_2 centroid body of K is established.

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