

COMPLETE CONSISTENCY AND CONVERGENCE RATE OF THE NEAREST NEIGHBOR ESTIMATOR OF THE DENSITY FUNCTION BASED ON WOD SAMPLES

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Abstract. By using the exponential inequality of widely orthant dependent (WOD, for short) random variables, we mainly investigate the complete consistency and convergence rate of the nearest neighbor estimator of the density function based on WOD samples. The results obtained in the paper generalize and improve some corresponding ones in the literature. In addition, the restriction on the dominating coefficients $g(n)$ is much weak, even if the geometric growth of $g(n)$, the consistency result and convergence rate still hold by using the results that we obtained.

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