

POLYA—VINOGRADOV INEQUALITY FOR POLYNOMIAL CHARACTER SUMS OVER FINITE FIELDS

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Abstract. A version of Polya–Vinogradov inequality in function fields appeared in [1, 2, 3, 9] recently. In this paper, we show some new bounds for polynomial character sums by making use of polynomial Gauss sums (see [4, 12]) and a formula from L. Carlitz (see [5]) on exponential sums over function fields. The method is elementary. It is worth mentioning that the proofs given in this paper do not depend on the well-known result from A. Weil on L-function associated to algebraic curves over finite fields.

Mathematics subject classification (2020): 11T55, 11T24.

Keywords and phrases: Polya–Vinogradov inequality, Gauss sums, character sums, function fields.

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