

NONLINEAR ELLIPTIC EQUATIONS WITH SMALL *BMO* COEFFICIENTS IN NONSMOOTH DOMAINS IN GENERALIZED MORREY SPACES

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Abstract. We obtain the generalized Sobolev-Morrey spaces $W_{p,\varphi}^1(\Omega)$ estimate for weak solutions of a boundary value problem for nonlinear elliptic equations with *BMO* coefficients in nonsmooth domains. We investigate regularity of the weak solutions in generalized Morrey spaces $M_{p,\varphi}(\Omega)$. The nonlinearity has sufficiently small *BMO* seminorm and the boundary of the domain is sufficiently flat.

Mathematics subject classification (2020): 35J25, 35B40, 42B20, 42B25.

Keywords and phrases: Nonlinear elliptic equations, generalized Sobolev-Morrey spaces, maximal function, Reifenberg flat domain, small *BMO*.

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