A NECESSARY AND SUFFICIENT CONDITION FOR THE
CONVEXITY OF THE ONE–PARAMETER GENERALIZED INVERSE
TRIGONOMETRIC SINE FUNCTION ACCORDING TO POWER MEAN

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Abstract. In the article, we present a necessary and sufficient condition such that the one-parameter generalized inverse trigonometric sine function is convex with respect to power mean. As a consequence, we provide the necessary and sufficient condition for the concavity of the one-parameter generalized trigonometric sine function according to power mean.

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