

OPTIMAL CONTROL FOR ELLIPTIC HEMIVARIATIONAL INEQUALITIES INVOLVING NONLINEAR WEAKLY CONTINUOUS OPERATORS

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Abstract. We study an optimal control problem governed by elliptic hemivariational inequalities involving nonlinear weakly continuous operators. By exploiting the surjectivity theorem of multivalued weakly upper continuous operators, we present an existence result for a class of subgradient inclusions involving nonlinear weakly continuous operators. Then we obtain the existence of optimal pairs for the optimal control problem. Moreover, we consider a perturbed optimal control problem and obtain the convergence of optimal pairs. This study can be applied to stationary Navier-Stokes problems with multivalued frictional boundary condition.

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