OPTIMAL BOUNDS FOR THE SÁNDOR MEAN IN TERMS OF THE COMBINATION OF GEOMETRIC AND ARITHMETIC MEANS

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Abstract. In this paper, we prove that \( \lambda = 1/2 - \sqrt{1 - e^{-2/p}/2} \) and \( \mu = 1/2 - \sqrt{6p/(6p)} \) are the best possible parameters on the interval \((0, 1/2)\) such that the double inequalities

\[
G^p [\lambda a + (1 - \lambda) b, \lambda b + (1 - \lambda) a] A^{1-p} (a, b) < X (a, b)
\]

\[
< G^p [\mu a + (1 - \mu) b, \mu b + (1 - \mu) a] A^{1-p} (a, b)
\]

hold for all \( p \in [1, \infty) \) and \( a, b > 0 \) with \( a \neq b \), where \( G(a, b) \) is the geometric mean, \( A(a, b) \) is the arithmetic mean, and \( X (a, b) \) is the Sándor mean.

Keywords and phrases: Sándor mean, geometric mean, arithmetic mean, inequality.

REFERENCES