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SHERMAN'S OPERATOR INEQUALITY

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Abstract. In this paper we deal with Sherman's inequality and its complementary inequalities for operator convex functions, whose arguments are the bounded self-adjoint operators from C^*-algebra on a Hilbert space and positive linear mappings between C^*-algebras. We introduce the so called Sherman's operator and study its properties. Applying an extended idea of convexity to operator functions of several variables, we obtain multidimensional Sherman's operator inequality. We define multidimensional Sherman's operator and study its properties. At the end, we observe applications to some operator inequalities related to connections, solidarities, and multidimensional weighted geometric mean.

Mathematics subject classification (2020): 47A63, 47A64.
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