

BOUNDEDNESS OF THE MARCINKIEWICZ INTEGRAL ON GRAND VARIABLE HERZ SPACES

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Abstract. We prove the boundedness of the Marcinkiewicz integral on grand variable Herz spaces.

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