

GENERALIZED HERMITE–HADAMARD TYPE INEQUALITIES FOR DIFFERENTIABLE HARMONICALLY–CONVEX AND HARMONICALLY QUASI–CONVEX FUNCTIONS

MUHAMMAD AMER LATIF, SABIR HUSSAIN AND YU-MING CHU*

Abstract. Some new Hermite–Hadamard type inequalities for differentiable harmonically-convex and harmonically quasi-convex functions have been discussed, generalizing some existing results in literature. For validity of the results some numerical examples are given.

Mathematics subject classification (2020): 26D15, 26D20, 26D07.

Keywords and phrases: Hermite–Hadamard’s inequality, harmonically-convex function, harmonically quasi-convex function, Hölder’s integral inequality.

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