

SOME MIXED WEAK TYPE INEQUALITIES

MARÍA LORENTE AND FRANCISCO J. MARTÍN-REYES

Abstract. We study mixed weak type (1,1) weighted inequalities for the Hardy averaging operator, $T_c f(x) = \chi_{(c,\infty)}(x) \frac{1}{x-c} \int_c^x f(y) dy$. This type of inequalities have the form

$$\int_{\{x \in \mathbb{R}: |T_c f(x)| > v(x)\}} uv \leq C \int_{\mathbb{R}} |f| u,$$

where C is independent of f and c . We improve the results in [Q. J. Math. 60 (2009), no. 1, 63–73] by giving a wider class of pairs of weights for which the inequality holds. In particular, and as a corollary, we prove that the inequality holds for $u \in A_1^-$ and $v \in A_\infty^+$.

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