

APPROXIMATION BY α -BERNSTEIN-SCHURER-STANCU OPERATORS

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Abstract. In this paper, we consider a new family of generalized Bernstein-Schurer-Stancu operators, depending on a non-negative real parameter α and study some approximation properties of these operators. We obtain a recurrence formula concerning calculation of moments by Schurer-Stancu operators. We prove a uniform approximation result using the well-known Korovkin theorem and obtain the rate of convergence in terms of modulus of continuity. Also, we present Voronovskaya and Grüss-Voronovskaya type results for these operators. Moreover, we give some numerical examples to illustrate approximation by the new operator.

Mathematics subject classification (2020): 41A10, 41A25, 41A36.

Keywords and phrases: Bernstein-Schurer-Stancu operators, α -Bernstein operator, modulus of continuity, Voronovskaya type theorem, Grüss-Voronovskaya type theorem.

REFERENCES

- [1] T. ACAR, A. M. ACU, N. MANAV, *Approximation of functions by genuine Bernstein-Durrmeyer type operators*, Journal of Mathematical Inequalities 2018; 12 (4): 975–987.
- [2] A. M. ACU, H. GONSKA, I. RAŞA, *Grüss-type and Ostrowski-type inequalities in approximation theory*, Ukrainian Mathematical Journal 2011; 63 (6): 843–864.
- [3] D. BARBOSU, *Schurer-Stancu type operators*, Studia Universitatis Babeş-Bolyai Mathematica, 2003; XLVIII (3): 31–35.
- [4] S. N. BERNSTEIN, *Démonstration du théorème de Weierstrass fondée sur le calcul des probabilités*, Communications Kharkov Mathematical Society 1912/1913; Vol. 13: 1–2.
- [5] J. BUSTAMANTE, *Bernstein Operators and Their Properties*, Springer International Publishing, Birkhäuser, 2017.
- [6] Q. CAI, X. XU, *Shape-preserving properties of a new family of generalized Bernstein operators*, Journal of Inequalities and Applications 2018; 2018:241.
- [7] N. ÇETIN, *Approximation and geometric properties of complex α -Bernstein operator*, Results in Mathematics 2019; 74: 40, <https://doi.org/10.1007/s00025-018-0953-z>.
- [8] N. ÇETIN, *Approximation by α -Bernstein-Schurer operator*, under review.
- [9] N. ÇETIN, V.A. RADU, *Approximation by generalized Bernstein–Stancu operators*, Turkish Journal of Mathematics 2019; 43: 2032–2048.
- [10] X. CHEN, J. TAN, Z. LIU, J. XIE, *Approximation of functions by a new family of generalized Bernstein operators*, Journal of Mathematical Analysis and Applications 2017; 450: 244–261.
- [11] S. GAL, H. GONSKA, *Grüss and Grüss-Voronovskaya-type estimates for some Bernstein-type polynomials of real and complex variables*, Jaen Journal on Approximation, 2015; 7 (1): 97–122.
- [12] H. GONSKA, G. TACHEV, *Grüss-type inequalities for positive linear operators with second order moduli*, Matematički Vesnik, 2011; 63 (4): 247–252.
- [13] V. GUPTA, T. M. RASSIAS, P. N. AGRAWAL, A. M. ACU, *Estimates for the Differences of Positive Linear Operators*, In: Recent Advances in Constructive Approximation Theory. Springer Optimization and Its Applications, vol 138, (2018), Springer, Cham.
- [14] S. A. MOHIUDDINE, T. ACAR, A. ALOTAIBI, *Construction of a new family of Bernstein-Kantorovich operators*, Mathematical Methods in the Applied Sciences 2017; 40: 7749–7759.
- [15] F. SCHURER, *Linear positive operators in approximation theory*, Math. Inst. Techn. Univ. Delft Report, 1962.

- [16] M. SMUC, *On a Chlodovsky variant of Bernstein operator*, Bulletin of the Transilvania University of Braşov, Series III: Mathematics, Informatics, Physics 2017; 10 (1): 165–178.
- [17] D. D. STANCU, *Asupra unei generalizări a polinoamelor lui Bernstein*, Studia Universitatis Babeş-Bolyai, Ser. Math.-Phys., 1969; 14 (2): 31–45 (in Romanian).