

## A COMPLETE CONVERGENCE THEOREM FOR WEIGHTED SUMS UNDER THE SUB-LINEAR EXPECTATIONS

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*Abstract.* In this article, we study a complete convergence theorem for weighted sums in sub-linear expectations space. We establish a complete convergence theorem for weighted sums under the optimal moment conditions in sub-linear expectations space. Our result extends and improves the corresponding result of Cai (Metrika, 68:323-331, 2008) in some extent.

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