COEFFICIENT ESTIMATES AND FEKETE–SZEGÖ INEQUALITY FOR NEW SUBCLASS OF BI–BAZILEVIČ FUNCTIONS BY (s,t)–DERIVATIVE OPERATOR AND QUASI–SUBORDINATION

AOEN*, SHUHAI LI AND HUO TANG

Abstract. In this paper we introduce and investigate a new generalized class of bi-bazilevič functions defined by using (s,t)-derivative operator and quasi-subordination in the open unit disk $\mathbb{D}$. We obtain two kinds of coefficient estimate by using Faber polynomial expansion and get Fekete–Szegö inequality for the new class and some of its subclasses.

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