

## NONLINEAR INEQUALITIES AND RELATED FIXED POINT PROBLEMS

MUHAMMAD NAZAM, ESKANDAR AMEER,  
MOHAMMAD MURSALEEN\* AND ÖZLEM ACAR

**Abstract.** In this paper, we introduce a nonlinear inequality based on four self-mappings. We give necessary conditions which ensure the existence of a common fixed point of four self-mappings satisfying said inequality defined in  $\mathcal{S}$ -metric spaces. A common fixed point problem is discussed. We set up an example to elucidate our main result. Moreover, the existence of a common solution to a system of four integral equations is shown by application of main result.

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### REFERENCES

- [1] O. ACAR, I. ALTUN, *Multivalued  $F$ -contractive mappings with a graph and some fixed point results*, Publ. Math. Debrecen **88** (2016), 305–317.
- [2] M. U. ALI, T. KAMRAN, *Multivalued  $F$ -contractions and related fixed point theorems with an application*, Filomat **30** (14) (2016), 3779–3793.
- [3] M. ASADI, P. SALIMI, *Some Fixed point and common fixed point theorems for  $G$ -metric spaces*, Nonlinear Funct. Anal. Appl., **21** (3) (2016), 523–530.
- [4] S. BANACH, *Sur les opérations dans les ensembles abstraits et leur application aux équations itératives*, Fund. Math. **3** (1922), 133–181.
- [5] S. CZERWIK, *Nonlinear set-valued contraction mappings in  $b$ -metric spaces*, Atti Semin. Mat. Fis. Univ. Modena. **46** (2) (1998), 263–276.
- [6] S. CZERWIK, *Contraction mappings in  $b$ -metric spaces*, Acta Math. Inf. Univ. Ostrav **1** (1993), 5–11.
- [7] B. C. DHAGE, *Generalized metric space and mapping with fixed point*, Bulletin of the Calcutta Mathematical Society **84** (1992), 329–336.
- [8] B. C. DHAGE, *On generalized metric spaces and topological structure II*, Pure and Applied Mathematics Sciences **40** (1994), 37–41.
- [9] G. DURMAZ, G. MINAK, I. ALTUN, *Fixed points of ordered  $F$ -contractions*, Hacettepe Journal of Mathematics and Statistics **45** (1) (2016), 15–21.
- [10] S. GAHLER, *2-metrische Räume und ihre topologische Struktur*, Mathematische Nachrichten **26** (1963), 115–148.
- [11] S. GAHLER, *Zur geometrische 2-metrische raume*, Revue Roumaine de Mathématiques Pures et Appliquées **11** (1966), 664–669.
- [12] A. JAVAHERI, S. SEDGHI, H. G. HYUN, *Common fixed point theorems for two mappings in  $S$ -metric spaces*, Nonlinear Funct. Anal. Appl., **24** (2) (2019), 417–425.
- [13] Z. MUSTAFA, B. SIMS, *A new approach to generalized metric spaces*, J. Nonlinear Convex Anal. **7** (2006), 289–297.
- [14] M. NAZAM, N. HUSSAIN, A. HUSSAIN, M. ARSHAD, *Fixed point theorems for weakly admissible pair of  $F$ -contractions with application*, Nonlinear Analysis: Modelling and Control **24** (6) (2019), 898–918.
- [15] H. PIRI, P. KUMAM, *Some fixed point theorems concerning  $F$ -contraction in complete metric spaces*, Fixed Point Theory Appl. **2014** (2014), 11 pages.

- [16] M. M. REZAEI, S. SEDGHI, K. S. KIM, *Coupled common fixed point results in ordered  $S$ -metric spaces*, *Nonlinear Func. Anal. Appl.*, **23** (3) (2018), 595–612.
- [17] S. SEDGHI, N. SHOBE, A. ALIOUCHE, *A generalization of fixed point theorem in  $S$ -metric spaces*, *Mat. Vesnik* **64** (2012), 258–266.
- [18] S. SEDGHI, N. SHOBE, H. ZHOU, *A common fixed point theorem in  $D^*$ -metric spaces*, *Fixed Point Theory Appl.* Vol. **2007**, Article ID 27906 (2007), 13 pages.
- [19] S. SEDGHI, A. GHOLIDAHNEH, K. P. R. RAO, *Common fixed point of two  $R$ -weakly commuting mappings in  $S_b$ -metric spaces*, *Math. Sci. Lett.* **6** (3) (2017), 249–253.
- [20] D. SINGH, V. JOSHI, J. K. KIM, *Existence of solution to Bessel-type boundary value problem via  $G$ - $I$  cyclic  $F$ -contractive mappings with graphical verification*, *Nonlinear Func. Anal. Appl.*, **23** (2) (2018), 205–224.
- [21] D. WARDOWSKI, *Fixed points of a new type of contractive mappings in complete metric spaces*, *Fixed Point Theory Appl.* **2012**, (2012): 6 pages.
- [22] M. YOUNIS, D. SINGH, D. GOPAL, A. GOYAL, M. S. RATHORE, *On applications of generalized  $F$ -contraction to differential equations*, *Nonlinear Func. Anal. Appl.*, **24** (1) (2019), 155–174.