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INTEGRALS OF RATIOS OF FOX–WRIGHT AND INCOMPLETE FOX–WRIGHT FUNCTIONS WITH APPLICATIONS

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Abstract. The main focus of the present paper is to establish definite integral formulae for ratios of the Fox–Wright functions. As consequences of the master formula, some novel integral formulae are derived for ratios of other special functions which are associated to Fox–Wright Ψ function, like generalized hypergeometric function, modified Bessel function of the first kind and Mittag–Leffler type functions of two and three parameters. Moreover, closed integral form expressions are obtained for a family of Mathieu-type series and for the associated alternating versions whose terms contain the incomplete Fox-Wright function. As applications, functional bounding inequalities are established for the aforementioned series.

Mathematics subject classification (2020): 26D15, 33C20, 33C70, 33E12, 40C10.
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