Journal of Mathematical Inequalities
Volume 15, Number 3 (2021), 1075–1091
doi:10.7153/jmi-2021-15-73

# STABILITY RESULT OF LAMINATED BEAM WITH INTERNAL DISTRIBUTED DELAY

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*Abstract.* In this paper, we consider a laminated Timoshenko beam system with frictional damping and an internal distributed delay feedback on the effective rotational angle. Under appropriate assumptions on the weight of the delay term and wave speeds of the first two equations of the system, we prove that the dissipation through the frictional damping is enough to stabilize the system exponentially.

*Mathematics subject classification* (2020): 26D10, 35L20, 74H55, 93D15.
*Keywords and phrases*: Laminated beams, frictional damping, distributed delay, exponential stability.

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