

## RECONSTRUCTION OF TWO APPROXIMATION PROCESSES IN ORDER TO REPRODUCE $e^{ax}$ AND $e^{2ax}$ , $a > 0$

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*Abstract.* We propose two modifications for Gauss-Weierstrass operators and moment-type operators which fix  $e^{ax}$  and  $e^{2ax}$  with  $a > 0$ . First, we present moment identities for new operators. Then, we discuss weighted approximation and prove Voronovskaya-type theorems for them in exponentially weighted spaces. Using modulus of continuity in exponentially weighted spaces, we obtain some global smoothness preservation properties. We give a comparison result for Gauss-Weierstrass operators. Finally, we provide some graphical illustrations that show that modified operators perform better than classical ones.

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