

## A CHAIN OF NUMERICAL RADIUS INEQUALITIES IN COMPLEX HILBERT SPACE

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**Abstract.** In this paper, we implement the improvement of numerical radius inequalities that were produced by Alomari MW. [Refinements of some numerical radius inequalities for Hilbert space operators. *Linear and Multilinear Algebra*. 2019 Jun 4:1-6] and devise a new upper bound for  $2 \times 2$  operator matrices on complex Hilbert space with many examples which show that our bound is sharper than the existing bounds proved by Bani-Domi W, Kittaneh F. [Norm and numerical radius inequalities for Hilbert space operators. *Linear and Multilinear Algebra*. 2020 Jul 28:1-2], Al-Dolat M, Jaradat I, Al-Husban B. A novel numerical radius upper bounds for  $2 \times 2$  operator matrices. *Linear and Multilinear Algebra*. 2020 Apr 23:1-2], Shebrawi K. [Numerical radius inequalities for certain  $2 \times 2$  operator matrices II. *Linear Algebra and its Applications*. 2017 Jun 15; 523:1-2] and Hirzallah O, Kittaneh F, Shebrawi K. [Numerical radius inequalities for  $2 \times 2$  operator matrices. *Studia Mathematica*. 2012; 210:99-115].

*Mathematics subject classification* (2020): 47A12, 47A30, 47A63, 47B33.

*Keywords and phrases:* Numerical radius, spectral radius, operator matrix.

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