Journal of Mathematical Inequalities
Volume 15, Number 3 (2021), 1233–1237
doi:10.7153/jmi-2021-15-82

NEW BOUNDS FOR THE RATIO OF POWER MEANS

ZSOLT PÁLES AND PAWEŁ PASTECZKA

Abstract. We show that for real numbers p, q with $q < p$, and the related power means $\mathscr{P}_p$, $\mathscr{P}_q$, the inequality

$$\frac{\mathscr{P}_p(x)}{\mathscr{P}_q(x)} \leqslant \exp\left(\frac{p-q}{8} \cdot \left(\ln\left(\frac{\max x}{\min x}\right)\right)^2\right)$$

holds for every vector x of positive reals. Moreover we prove that, for all such pairs (p,q), the constant $\frac{p-q}{8}$ is sharp.

Mathematics subject classification (2020): 26E60, 26D15, 26D07.
Keywords and phrases: Hölder means, power means, ratio bounds.

REFERENCES

[1] J. M. BORWEIN AND P. B. BORWEIN, *Pi and the AGM*, Canadian Mathematical Society Series of Monographs and Advanced Texts, John Wiley & Sons, Inc., New York, 1987. A study in analytic number theory and computational complexity, A Wiley-Interscience Publication.
[2] G. T. CARGO AND O. SHISHA, *Bounds on Ratios of Means*, Journal of research of the National Bureau of Standards-B, Mathematics and Mathematical Physics, 66B (4): 169–170, 1962.
[3] L. V. KANTOROVICH, *Functional analysis and applied mathematics*, Uspekhi Mat., 28 (6): 89–185, 1948.