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ON A HILBERT–TYPE INEQUALITY WITH THE KERNEL INVOLVING EXTENDED HARDY OPERATOR

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Abstract. In this paper by defining a extended Hardy operator, a new kernel function including both the homogeneous and the non-homogeneous cases is constructed. Dealing with these cases in a unified way, a Hilbert-type inequality involving the newly constructed kernel is established, and the constant factor is proved to be the best possible. The equivalent Hardy-type inequality is also considered in parallel. Furthermore, by specifying the kernel function, some special and meaningful Hilbert-type inequalities with the constant factors related to the higher derivative of trigonometric functions and special functions are presented at the end of the paper, and these newly obtained inequalities are proved to be the extensions of some classical Hilbert-type inequalities.

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