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ON THE ARITHMETIC–GEOMETRIC MEAN INEQUALITY

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Abstract. In this article, we present a new treatment of the arithmetic-geometric mean inequality and its siblings, the Heinz and the Young inequalities. New refinements via calculus computations and convex analysis are presented and a new Heinz-type inequality is presented for any symmetric operator mean.

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