ABSOLUTELY MONOTONIC FUNCTIONS INVOLVING THE COMPLETE ELLIPTIC INTEGRALS OF THE FIRST KIND WITH APPLICATIONS

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Abstract. Let \( K(r) \) be the complete elliptic integral of the first kind. In this paper, we prove that the function \( F_p(x) = (1 - x)^p \exp K(\sqrt{x}) \) is absolutely monotonic on \((0, 1)\) if and only if \( p \leq \pi/8 \), and \( -F'_p(x) \) is absolutely monotonic on \((0, 1)\) if and only if \( 1/2 \leq p \leq (\pi + 4 + \sqrt{16 - \pi})/8 \). This generalizes a known result and gives several new inequalities involving the complete elliptic integral of the first kind.

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REFERENCES


