SINGULAR VALUE AND NORM
INEQUALITIES OF DAVIDSON–POWER TYPE

Wasim Audeh

Abstract. Let A, B, X and Y be $n \times n$ complex matrices such that A and B are positive semidefinite, then

$$\|AX + YB\| \leqslant \frac{1}{4} \left(\|W_1\| + \|W_2\| + W_4 \right),$$

where

$$W_1 = A + A^{1/2} |X^*|^2 A^{1/2},$$
$$W_2 = B + B^{1/2} |Y|^2 B^{1/2},$$
$$W_3 = A^{1/2} X B^{1/2} + A^{1/2} Y B^{1/2}$$

and

$$W_4 = \sqrt{(\|W_1\| - \|W_2\|)^2 + 4\|W_3\|^2}.$$

Multiple results are given in this paper.

Mathematics subject classification (2020): 15A18, 15A42, 15A60, 47A30, 47B15.

Keywords and phrases: Concave function, positive semidefinite matrix, singular value, unitarily invariant norm, inequality.

REFERENCES

[1] W. Audeh, *Some generalizations for singular value inequalities of compact operators*, Adv. Oper. Theory 6 (2021).
[2] W. Audeh, *Singular value inequalities and applications*, Positivity 25 (2020), 843–852.
[3] W. Audeh, *Generalizations for singular value and arithmetic-geometric mean inequalities of operators*, J. Math. Anal. Appl. 489 (2020), 1–8.
[4] W. Audeh, *Generalizations for singular value inequalities of operators*, Adv. Oper. Theory 5 (2020), 371–381.
[5] W. Audeh, F. Kittaneh, *Singular value inequalities for compact operators*, Linear Algebra Appl. 437 (2012), 2516–2522.
[6] R. Bhatia, *Matrix Analysis*, Springer, New York, 1997.
[7] R. Bhatia and F. Kittaneh, *On the singular values of a product of operators*, SIAM J. Matrix Anal. Appl. 11 (1990), 272–277.
[8] J. C. Bourin, *A matrix subadditivity inequality for symmetric norms*, Proc. Amer. Math. Soc. 138 (2009), 495–504.
[9] K. Davidson, S. C. Power, *Best approximation in C^*-algebras*, J. Reine Angew. Math. 368 (1986), 43–62.
[10] O. Hirzallah, *Inequalities for sums and products of operators*, Linear Algebra Appl. 407 (2005), 32–42.
[11] R. A. Horn, and C. R. Johnson, *Matrix Analysis*, 2nd edition, Cambridge University Press, 2013.
[12] J. C. Hou, H. K. Du, *Norm inequalities of positive operator matrices*, Integral equations operator theory 22 (1995), 281–294.
[13] F. Kittaneh, *Norm inequalities for certain operator sums*, J. Funct. Anal. 143 (1997), 337–348.
[14] F. Kittaneh, *Norm inequalities for sums of positive operators*, J. Operator theory 48 (2002), 95–103.

[15] X. ZHAN, *Singular values of differences of positive semidefinite matrices*, SIAM J. Matrix Anal. Appl. 22 (2002), 819–823.

Journal of Mathematical Inequalities
www.ele-math.com
jmi@ele-math.com