

ON THE GENERALIZED QUADRATIC GAUSS SUMS AND ITS UPPER BOUND ESTIMATE

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Abstract. The main purpose of this paper is to study generalized quadratic Gauss sums, then use the analytic methods, the properties of the classical Gauss sums and character sums to give a sharp upper bound estimate for it. In addition, we also give several interesting fourth and sixth power mean formulae for the sums.

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