GENERALIZATION AND REFINEMENTS OF THE JENSEN–MERCER INEQUALITY WITH APPLICATIONS

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Abstract. We give a generalization followed by refinements of Jensen-Mercer inequality in variety of ways. We also highlight its importance by stating plenty of applications. In this way our main results generalize many established results including Ky Fan's Inequality, Popoviciu's inequalities and Rado's inequalities etc.

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