

## PROOF OF A DOUBLE INEQUALITY IN TRIANGLES

JIAN LIU

**Abstract.** A double geometric inequality involving the side lengths, medians, angle bisectors and exradius of a triangle is proved by applying the “ $R - r - s$ ” method in the theory of triangle inequalities. Several corollaries are obtained by using the main result and the other known inequalities.

**Mathematics subject classification (2020):** 51M16.

**Keywords and phrases:** Triangle, Euler’s inequality, Gerretsen’s inequality, Sondat’s inequality, Ciamberlini’s inequality, Walker’s inequality.

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